https://doi.org/10.37442/fme.2025.1.73

Heat Transfer Modeling in Sweetened Condensed Milk Under Ambient Conditions

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ABSTRACT

Introduction: Sweetened condensed whole milk (SCM) is a highly demanded product among consumers and processing plants. The study of temperature profiles is essential for optimizing its logistics under extreme temperature conditions to prevent quality defects and minimize costs associated with specialized transport when delivering to regions with hot climates and the Far North, considering the absolute temperature range in Russia, which is approximately 90°C. Previously, this aspect of transportation had not been studied. The maximum allowable storage temperature for the product was set at 25°C, while the minimum temperature was not regulated.

Purpose: To investigate the temperature profiles of SCM in transport packaging under various ambient conditions.

Materials and Methods: A simulation of the heating and cooling processes of SCM in transport packaging, modeled as a one-dimensional multilayer system, was conducted. To describe heat transfer within the temperature ranges of 5°C to 35°C and 5°C to -35°C, a system of differential equations was formulated, with specified initial and boundary conditions.

Results: According to the proposed model, the duration of heating SCM from 5°C to 35°C is 36.7 hours, while cooling from 5°C to -35°C takes 41.1 hours. Based on the study results, software was developed to calculate the duration of SCM temperature changes depending on the initial and final ambient temperatures.

Conclusion: A new approach has been developed for theoretically predicting the duration of temperature changes in SCM within transport packaging during storage and transportation. This approach can be utilized in specialized business software solutions for logistics route planning, transportation cost estimation, and consideration of ambient conditions during shipping. Additionally, the proposed solution can be adapted for other food products.

Keywords: sweetened condensed milk; heat transfer; one-dimensional heat conduction equation; air properties in the boundary layer

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Conflict of interest:
The authors report the absence of a conflict of interest.

Received: 10.11.2024
Accepted: 15.03.2025
Published: 31.03.2025

To cite: Bolshakova E.I., Motylev S.V., Semipyatny V.C., Kruchinin A.G., Turovskaya S.N., & Illarionova E.E. (2025). Heat transfer modeling in sweetened condensed milk. *FOOD METAENGINEERING*, *3*(1), 56-72. https://doi.org/10.37442/fme.2025.1.73

https://doi.org/10.37442/fme.2025.1.73

Моделирование процесса теплообмена с окружающей средой сгущенного молока с сахаром

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АННОТАЦИЯ

Введение: Сгущенное цельное молоко с сахаром (СЦМС) — востребованный продукт среди потребителей и переработчиков. Исследование температурных профилей необходимо для оптимизации его логистики в условиях экстремальных температур, чтобы предотвратить пороки качества и минимизировать затраты на специализированный транспорт при поставках в регионы с жарким климатом и районы Крайнего Севера, с учетом абсолютного температурного диапазона в РФ, составляющего ~90°С. Ранее данный вопрос в аспекте транспортирования не был исследован, максимально допустимая температура хранения для продукта составляла 25°С, минимальная не регламентировалась.

Цель: Исследование температурных профилей СЦМС в транспортной упаковке при различных условиях окружающей среды.

Материалы и методы: Осуществлено моделирование процессов нагревания и охлаждения СЦМС в транспортной упаковке, представленной в виде одномерной многослойной системы. Для описания задачи теплопередачи в температурных диапазонах от 5°С до 35°С и от 5°С до минус 35°С составлена система дифференциальных уравнений, определены начальные и граничные условия.

Результаты: Согласно построенной модели, продолжительность прогрева СЦМС от 5°С до 35°С составит 36,7 ч, а охлаждения от 5°С до минус 35°С — 41,1 ч. По результатам исследования разработано программное обеспечение для расчета продолжительности изменения температуры СЦМС в зависимости от начальных и конечных температур окружающей среды.

Выводы: Разработан новый подход к теоретическому прогнозированию продолжительности изменения температуры СЦМС в транспортной упаковке при хранении и транспортировании. Данный подход может быть использован в специальных программных обеспечениях для бизнеса при планировании логистических маршрутов, затрат на транспортирование с учетом срока перевозки и условий окружающей среды. Также, предложенное решение может быть адаптировано под другие пищевые продукты.

Ключевые слова: сгущенное молоко с сахаром; теплопередача; одномерное уравнение теплопроводности; свойства воздуха в граничном слое

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Конфликт интересов: авторы сообщают об отсутствии конфликта интересов.

Поступила: 10.11.2024
Принята: 15.03.2025
Опубликована: 31.03.2025

Для цитирования: Большакова, Е.И., Мотылев, С.В., Семипятный, В.К., Кручинин, А.Г., Туровская, С.Н., & Илларионова, Е.Е. (2025). Моделирование процесса теплообмена с окружающей средой сгущенного молока с сахаром. *FOOD METAENGINEERING*, *3*(1), 56-72. https://doi.org/10.37442/fme.2025.1.73

INTRODUCTION

At various stages of production and sale of a product, there are a number of factors affecting its quality: the initial properties and characteristics of raw material, proper execution of technological processes, proper operation of equipment, ambient conditions during transportation and storage (Sharma et al., 2015). Canned food is a group of products that retain quality for a long shelf life, since their production uses modifications of the principles of suspended anabiosis, namely thermoanabiosis, xeroanabiosis and osmoanabiosis. This fact plays a great role in the reason that canned food is a convenient and preferred commodity for export. To date, canned milk, and in particular sweetened condensed whole milk, is not only a product of the daily diet of consumers, but also material for production of other dairy, confectionery and bakery products (Ryabova et al., 2022). According to analytical resources, for 2023, domestic exports of condensed milk increased by USD 6.5 million in only one direction, as well as new manufacturers-exporters of SCM appeared[1,2].

It should be noted that due to the changing geopolitical and economic situation, there is a transformation of logistics supply chains, which allows not only to expand territories for exports, but also poses a number of tasks for manufacturers (Ledneva et al., 2023; Turovskaya et al., 2024). It is known, for example, that according to GOST 31688–2012 SCM can be transported by all means of transport at a maximum permissible temperature of 25°C, which is specified in the standard technological instructions for this GOST. To comply with this requirement, when transporting SCM to Asian and African countries, it is necessary to use specialized transport to maintain a regulated temperature regime, which is expensive and constraints the capabilities of many enterprises (LeBlanc, 2005; Vanek and Sun, 2008; Singh and Negi, 2018). In this regard, it is relevant to study changes in the temperature profile of the SCM in uncontrolled temperature storage conditions, which will allow to extract data on both the duration of temperature equalization of the product and the temperature of the product at any point at a given time.

Experimentally, the effect of various storage modes on the properties of concentrated dairy products, SCM and its model systems has been studied by a number of authors. Thus, Galstyan et al. noted that the transportation of whole milk powder at temperatures above 10°C, as well as its freezing, can provoke degradation of protein, fat fraction of the product and cause structural changes in lactose (Galstyan et al., 2019). Ryabova et al. (2022) studied the change in the criteria for water crystallization in milk concentrated systems under research depending on concentration and temperature exposure (Ryabova, 2023), and also presented data on the results of differential scanning calorimetry of SCM. The researchers recorded the values of the freezing point, the mass fraction of moisture that has passed into

the frozen state, the glass transition temperature and the enthalpy of fusion, which may be important in modeling the processes of predicting changes in the quality of SCM at low negative temperatures (Ryabova et al., 2022; Ryabova, 2023). Patel et al. (1996) have created an approach to predicting storage capacity based on changes in the color of sweet concentrated milk according to its optical density.

Color change is not the only defect that can develop in condensed dairy products when stored in different temperature ranges. For example, Sharma et al. note that in addition to darkening of sweetened condensed milk, the most common defects in the product are thickening, loss of uniformity, sandiness, rancidity and metallic flavor (Sharma et al., 2015). In the research of Illarionova et al. (2020), an approach to predicting the thickening of the SCM is presented, which consists in holding the product at 98–100°C for 15–20 minutes and fixing the change in dynamic viscosity. Loss of fluidity after such exposure is accepted as a reason for refusing to send this batch of product for long-term storage. Appearing of thickening and other defects is due to the course of biochemical and microbiological processes (Illarionova et al., 2020), which in turn can be accelerated by changes in storage temperature regimes. At the same time, the heat load on the product is determined not only by the temperature, but also by the duration of storage at this temperature (Fox et al., 2015), which in turn raises interest in assessing the dynamics of changes in product temperature under various conditions.

[1] Roif Expert. (2023). Condensed milk market in Russia — 2023 exports amounted to + 6.5 million rubles. $ in the Kazakh direction. VC.ru. https://vc.ru/u/406653-roif-expert/636061-rynok-sgushchennogo-moloka-v-rossii-2023-eksport-sostavil-6–5-mln-po-kazahskomu-napravleniyu

[2] Dairy News. (2023). Vologda region exported more than 90 tons of condensed milk. https://dairynews.ru/news/vologodskaya-oblast-otpravila-na-eksport-bolee-90-.html

Various approaches are used to study the heat exchange process of a product under ambient conditions, with a primary focus on evaluating the efficiency of heat treatment to ensure its safety (Derossi et al., 2012; Kızıltaş et al., 2010). For example, researchers from the USA have validated the method of analytical and predictive numerical solution in two-dimensional computational domain (2D APNS method) to simulate product temperature profile at the slowest heating point of a tin can during retort processing (Zhu et al., 2022). The paper considered the limitations of the APNS method in one-dimensional space and proposed 2D APNS, which was used to simulate the temperature profiles of canned food under various heat transfer modes and taking into account several shapes of containers in which canned food was stored. The researchers validated this method by comparing it with experimental data.

In the issue of high-temperature processing, Paul et al. (2011) studied the process of temperature change during pasteurization of canned milk in a jar in two positions and compared with the results of a theoretical model of temperature distribution for the slowest heating point obtained using computational fluid dynamics. In turn, for the zone of negative temperatures Ryabova et al. (2023) developed a program for calculating the cooling time of a can of condensed milk, which allows to determine the time required for cooling and freezing a single package with the product. The researchers took convection into account by adjusting the heat transfer coefficient.

At the same time, despite the significance and fundamental nature of the presented research, the nature of temperature changes in the product in consumer packaging and in group or transport packaging will differ significantly, since the geometry of the object under study will change, which significantly affects the boundary conditions in the systems of equations to be solved. Thus, the purpose of this research is to solve the model problem of ambient heat exchange with SCM in transport packaging. The study aims to obtain data on the duration of temperature equalization of the product under ambient conditions and its temperature at any point at a given time to control and prevent risks of product quality changes during transportation. The paper considers a temperature range of ambient conditions from minus 35 to 35°C without considering humidity and temperature variation associated with solar radiation.

MATERIALS AND METHODS

Objects

The primary focus of this study is the heat transfer model of SCM in transport packaging under ambient conditions. The modeled transport packaging consists of 57 corrugated cartons stacked in six rows, each containing SCM packed in No. 7 metal cans, with 45 cans per carton. The parameters of the transport packaging, corrugated carton, and metal can are provided in Table 1.

Table 1
Parameters of the Objects Considered

Object	Typical dimensions, mm
Metal tin can No. 7	Thickness of metal layer: 0.25
Corrugated box	Height of box: 245, thickness of corrugation: 4
Transport packaging	Height: 1492

The modeled objects are combined into a multilayer system with three types of layers: (1) corrugated carton, (2) SCM, and (3) wooden base. The parameters of the multilayer system are provided in Table 2.

Table 2
Parameters of Multilayer System

Parameters	Values
Height of first layer, mm	4.0
Height of second layer, mm	237.0
Height of third layer, mm	22.0
Overall height	1492.0

As a first approximation, the heat transfer properties of the layers in the model are taken as average values (Table 3), without accounting for their temperature dependence.

Table 3
Heat transfer properties

№ of layer	ρ, kg/m³	c_p, J/kg	λ, W/(mK)	α, м²/c (·10E⁻⁵)
Layer 1	122	1150.0	0.070	0.049
Layer 2	1290	2260.9	0.267	0.009
Layer 3	500	1550.0	0.150	0.019

Where **α** is the coefficient of thermal diffusivity, ρ — density, c_p — specific isobaric heat capacity, λ — thermal conductivity.

Problem Statement

The product is delivered to the destination under ambient conditions with air temperatures ranging from -35°C to 35°C. The initial temperature of SCM in transport packaging is 5°C. It is necessary to calculate the time required for SCM to reach heat equilibrium with the ambient air under various conditions and to obtain data on the temperature of SCM at any position within the transport packaging at a given moment. The calculations in this study were performed using the parameters of the objects specified in Table 1.

Modeling and Mathematical Framework

Modeling at each stage consisted of two operations: 1 — analysis of objects (characteristic sizes, heat transfer properties) and their interaction in the transport packaging, 2 — introduction of assumptions based on the principle of estimating the minimum and maximum possible contribution to the change in the heat transfer properties of the SCM in the process of temperature change. To simplify the description of the model, the following designations are introduced:

Sc1 — area of contact of the side wall of the box with the side wall of the jar,

Sc2 — area of contact of the side walls of two cans,

Sc3 — area of contact of the lids and bottoms of cans with the upper and lower surfaces of the box.

The mathematical formulation of the problem was carried out using systems of differential equations describing heat processes, initial and boundary heat conditions. At this stage, the mathematical description was simplified, taking into account the assumptions introduced when compiling the physical model.

The analytical solution was carried out using the approach presented in (Biswas and Singh, 2015) with adaptation to the conditions of heat exchange of the SCM with the ambient air. The calculation of the equations is done using software developed in Wolfram Mathematica.

RESULTS AND DISCUSSION

Modeling

The study researches the temperature change of the SCM packed in metal cans No. 7, placed in a transport packaging. The thermal conductivity coefficient (λ) of metal from which the can is made is 47...52 W/(mK), which is many times higher than λ SCM (0.267 W/(mK). This fact indicates that the rate of heat transfer through the wall of a metal tin can will be much higher than the rate of heat transfer in SCM and causes the exclusion of metal from the designed model.

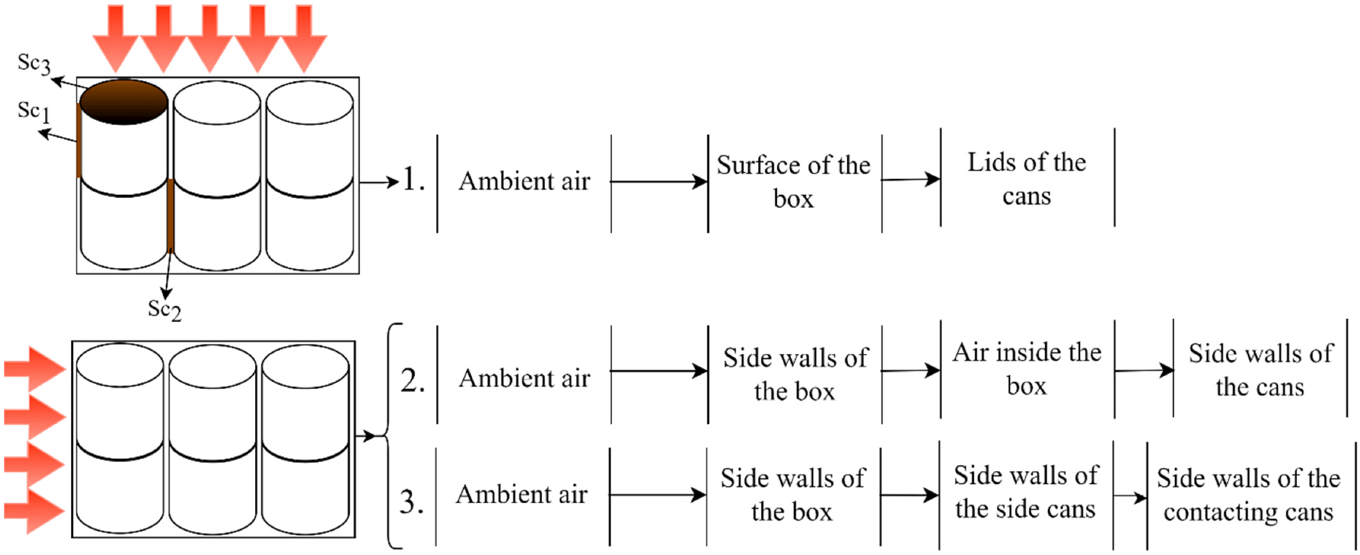
The SCM is in grouped packaging — a corrugated box (hereinafter referred to as the box), the inner volume of empty space is filled with air between the cans. Cans with SCM are tightly packed in a box in 3 rows wide, 5 rows length and 3 rows high. The fit of the side wall of one can to the side wall of another can inside the box; the side wall of the can to the side wall of the box; the bottom / lid of one can with sides with the bottom / lid of another can; lids / bottoms with the upper and lower surface of the box will be considered dense. As a result of the assumptions introduced in the model, there is an ideal heat contact between all contacting surfaces.

Heat transfer to the SCM can under ambient conditions occurs through the side, upper, and lower surfaces of the box (Figure 1).

It was decided to neglect the first approximation of heat transfer from the side surfaces of the box to the air inside the box and from the air inside the box to the side walls of the cans (Figure 1, scheme 2), since λ of air is an order of magnitude lower than the λ SCM, and accordingly the heat flow according to scheme 2 will be much lower compared to the heat flow according to scheme 1. Let's make a comparative assessment of the contribution of heat flows according to diagrams 1 and 3 (Figure 1–3). Since the heat flow is proportional to the area (Q~S), and Sc1 is less relative to Sc3, then with direct heat transfer according to diagram 3, the amount of heat transferred by the SCM through the side surface — Q1 will be lower than Q2 — the amount of heat transferred according to diagram 1 through the upper and lower surfaces of the box. Also, since Sc2 is significantly lower than Sc3, it was decided to neglect the heat exchange between the contacting cans through the side surface according to diagram 3 in the model. In view of this, we will assume that the heat

Figure 1

Schemes of Heat Transfer in SCM under Ambient Conditions



flow is carried out according to diagram 1 (Figure 1), it is one–dimensional and directed perpendicular to the upper and lower surfaces of the boxes, and the side wall of the box and the side walls of the cans are thermally insulated (adiabatic).

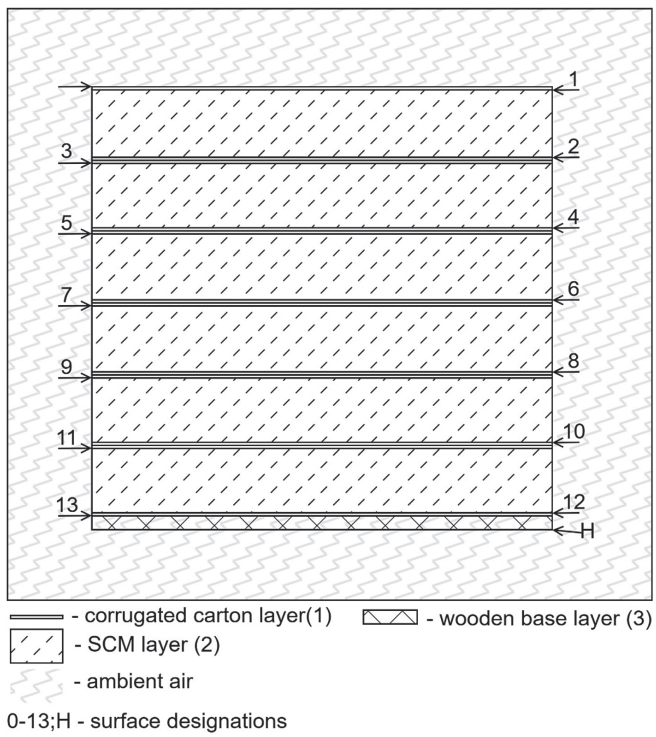
Transport packaging which is a rectangular parallelepiped consisting of 6 rows of boxes with SCM in height (evenly stacked and tightly touching the side walls) is placed on a wooden pallet. The upper surface of the pallet consists of 5 wooden boards and includes narrow air cavities (the characteristic size of the air cavities is much smaller than the width of the boards), in this regard, it was decided to take into account the pallet in the form of a dense wooden plate with a thickness of 22 mm, excluding the supporting wooden parts of the pallet.

Transport packaging placed on a wooden base will be considered similar to a grouped package in the form of a single box with SCM, i.e. for transport packaging a one-dimensionality of the heat flow is allowed. Thus, the entire transport packaging can be represented in the form of a one-dimensional multilayer system (Figure 2), in which the 1st layer is corrugated cardboard, the 2nd layer is SCM, and the 3rd layer is a wooden base.

The transport packaging is placed in a dry cargo container (hereinafter referred to as the container), which is filled with air. The container is made of steel, a material with

Figure 2

A Model Representation of a Transport Packaging with SCM in the Form of a Multilayer System to Study Heat Transfer



high thermal conductivity (λ steel at 20°C=40...50 W/(mK); λ air ≈ 0.025 W/(mK)), which provides good heat exchange with ambient air. This fact allows to assume that the air temperature inside the container is equal to the ambient temperature, since the rate of heat transfer through the walls of the container is higher than the rate of temperature change in the transport packaging. In this regard, further in the modeling, the air temperature (Ta) inside the container is assumed to be equal to the ambient temperature and unchanged, as well as other heat transfer properties of the air in the container. They are not affected by heat exchange processes with the transport packaging and SCM.

The heat exchange of SCM in transport packaging under ambient conditions was considered free convection, as there is no forced movement of air. It is accepted that at the initial time the transport packaging with SMS has a certain temperature in the entire volume.

In the "convective" layer—the part of the air where heat exchange occurs—the air has its own set of heat transfer properties, which depend on the temperature of the surface of the studied system of objects. These characteristics were calculated based on the table values presented in Lienhard (2019), using polynomials that best approximate them (Table 4–7).

Volume Density (ρ)

A calculation was performed for two equations (selecting coefficients of polynomials) and a comparison with ref-

Table 4

Determination of the Optimal Equation for Calculating Volume Density

Equation №1 – ρ = 353.089/T Type – hyperbolic regression				Equation №2 – ρ = –0.005491 + 354.683249/T Type – hyperbolic regression			
T, K	Estimate	Reference	ε, %	T, K	Estimate	Reference	ε, %
	ρ, kg/m³				ρ, кг/м³		
250	1,4124	1,4120	0,0252	250	1,4132	1,4120	0,0879
260	1,3580	1,3580	0,0025	260	1,3587	1,3580	0,0497
270	1,3077	1,3080	0,0201	270	1,3082	1,3080	0,0115
280	1,2610	1,2610	0,0025	280	1,2612	1,2610	0,0186
290	1,2175	1,2170	0,0451	290	1,2176	1,2170	0,0455
300	1,1770	1,1770	0,0031	300	1,1768	1,1770	0,0182
310	1,1390	1,1390	0,0003	310	1,1386	1,1390	0,0309
Average approximation error			0,0141	Average approximation error			0,0375

Table 5

Determination of the Optimal Equation for Calculating the Specific Isobaric Heat Capacity

Specific isobaric heat capacity (c_p)

Equation №1 — $c_p = 0.000370 \cdot T^2 - 0.187343 \cdot T + 1029.668537$ Type — quadratic regression				Equation №2 — $c_p = 0.019256 \cdot T + 1001.466942$ Type — linear regression			
T,K	Estimate	Reference	ε, %	T,K	Estimate	Reference	ε, %
	c_p, J/(kgK)				c_p, J/(kgK)		
250	1006.0	1006.0	0.004	250	1006.3	1006.0	0.028
260	1006.0	1006.0	0.003	260	1006.5	1006.0	0.047
270	1006.1	1006.0	0.006	270	1006.7	1006.0	0.066
280	1006.2	1006.0	0.022	280	1006.9	1006.0	0.085
290	1006.5	1006.0	0.045	290	1007.1	1006.0	0.104
300	1006.8	1007.0	0.023	300	1007.2	1007.0	0.024
310	1007.1	1007.0	0.015	310	1007.4	1007.0	0.043
Average approximation error			0.017	Average approximation error			0.057

erence values, an approximation error was calculated, on the basis of which the equation was selected for further calculations.

Equation No. 1 is used for further calculations of the volume density of air.

For further calculations of the specific isobaric heat capacity of air, equation No. 1 is used.

For further calculations of the dynamic air viscosity coefficient, equation No. 1 was used, since the value of the average approximation error for the first equation is greater due to the emission for a temperature of 310 K.

Table 6

Determination of the Optimal Equation for Calculating the Dynamic Viscosity Coefficient

Dynamic Viscosity Coefficient (μ)

Equation №1 — $\mu = 0.000000043 \cdot x^{3-} 0.000038698 \cdot x^{2} + 0.016251762 \cdot x - 0.714935371$ Type — cubic regression				Equation №2 — $\mu = — 0.00000311 \cdot T^2 + 0.00665672 \cdot T + 0.12896873$ Type — quadratic regression			
T, K	Estimate	Reference	ε, %	T, K	Estimate	Reference	ε, %
	μ,·10⁻⁵, Pa·s				μ, ·10⁻⁵, Pa·s		
250	1.601	1.606	0.295	250	1.599	1.606	0.450
260	1.650	1.649	0.079	260	1.649	1.649	0.029
270	1.698	1.699	0.040	270	1.700	1.699	0.033
280	1.746	1.747	0.082	280	1.749	1.747	0.116
290	1.792	1.795	0.150	290	1.798	1.795	0.160
300	1.839	1.857	0.982	300	1.846	1.857	0.588
310	1.885	1.889	0.199	310	1.894	1.889	0.248
Average approximation error			0.261	Average approximation error			0.232

Table 7

Determination of the Optimal Equation for Calculating the Coefficient of Thermal Conductivity

Coefficient of Thermal Conductivity (λ)

Equation №1 — $\lambda = 0.000065 \cdot x^3 - 0.059624 \cdot x^2 + 25.158592 \cdot x - 1336.027099$ Type — cubic regression				Equation №2 — $\lambda = -0.005622 \cdot x^2 + 10.598080 \cdot x - 55.398903$ Type — quadratic regression			
T. K	Estimate	Reference	ε, %	T, K	Estimate	Reference	ε, %
	λ				λ		
250	0.02243	0.02241	0.08	250	0.02243	0.02241	0.08
260	0.02317	0.02329	0.51	260	0.02323	0.02329	0.38
270	0.02390	0.02400	0.43	270	0.02396	0.02400	0.16
280	0.02461	0.02473	0.50	280	0.02471	0.02473	0.07
290	0.02531	0.02544	0.52	290	0.02545	0.02545	0.05
300	0.02600	0.02623	0.86	300	0.02618	0.02618	0.19
310	0.02670	0.02684	0.53	310	0.02690	0.02684	0.21
Average approximation error			0.49	Average approximation error			0.16

Equation No. 2 is used for further calculations of the air coefficient of thermal conductivity.

The heat transfer properties of air for each temperature are calculated using the polynomials presented above. According to Lienhard (2019), Churchill et al. (1975) the property of the medium (air in the problem) should be evaluated at the so-called "determining temperature" T_{det}

$$T_{det} = \frac{T_a + T_p}{2}, \tag{1}$$

Where T_p is the temperature of the product at the initial time; T_a is air temperature.

Thus, the following heat transfer characteristics were obtained for each discrete temperature value (Table 8).

Viscosity kinematic coefficient was calculated using the equation:

$$\nu = \frac{\mu}{\rho}. \tag{2}$$

Volumetric (Cubic) thermal expansion coefficient:

$$\beta = \frac{1}{T}. \tag{3}$$

Thermal diffusivity coefficient:

$$a = \frac{\lambda}{c_p \cdot \rho}. \tag{4}$$

The main difficulty of the calculation was to determine the convective heat transfer coefficient (α), which is included in the Newton-Richman boundary conditions for convective heat transfer.

The similarity theory of physical processes is usually used to calculate this coefficient. Its main idea is that processes of the same physical nature, which are characterized by the same mathematical description (differential equations, boundary and initial conditions) and the same geometry should proceed in a similar way. According to similarity theory, such qualitatively identical processes have equal similarity criteria and functional relationships between

Table 8

Heat Transfer Properties of Dry Air at a Pressure of 101325 Pa

T_a, (°C)	T_p, (°C)	T_{det}, (°C)	T_{det}, (K)	ρ, (kg/m³)	c_p, J/(kg·K)	μ(·10⁻⁵), Pa·s	λ, W/(mK)	ν (·10⁻⁵), m²/c	a (·10⁻⁵, m²/c)	β, 1/K	Pr
-35	5	-15.0	258.15	1.37	1005.96	1.64	0.023	1.20	1.68	0.0039	0.72
-30	5	-12.5	260.65	1.35	1005.97	1.65	0.023	1.22	1.71	0.0038	0.72
-25	5	-10.0	263.15	1.34	1005.99	1.67	0.023	1.24	1.74	0.0038	0.71
-20	5	-7.5	265.65	1.33	1006.01	1.68	0.024	1.26	1.77	0.0038	0.71
-15	5	-5.0	268.15	1.32	1006.04	1.69	0.024	1.28	1.80	0.0037	0.71
-10	5	-2.5	270.65	1.30	1006.07	1.70	0.024	1.30	1.83	0.0037	0.71
-5	5	0.0	273.15	1.29	1006.10	1.71	0.024	1.33	1.86	0.0037	0.71
0	5	2.5	275.65	1.28	1006.14	1.73	0.024	1.35	1.89	0.0036	0.71
5	5	5.0	278.15	1.27	1006.19	1.74	0.024	1.37	1.92	0.0036	0.71
10	5	7.5	280.65	1.26	1006.23	1.75	0.025	1.39	1.96	0.0036	0.71
15	5	10.0	283.15	1.25	1006.29	1.76	0.025	1.41	1.99	0.0035	0.71
20	5	12.5	285.65	1.24	1006.34	1.77	0.025	1.43	2.02	0.0035	0.71
25	5	15.0	288.15	1.23	1006.41	1.78	0.025	1.46	2.05	0.0035	0.71
30	5	17.5	290.65	1.21	1006.47	1.80	0.026	1.48	2.09	0.0034	0.71
35	5	20.0	293.15	1.20	1006.55	1.81	0.026	1.50	2.12	0.0034	0.71

them. This makes it possible to apply similarity criteria for a specific task using the similarity criteria obtained experimentally for modeling.

To describe the processes of free convection, the following similarity numbers are used:

Pr: Prandtl number; Nu: Nussel number; Ra:Rayleigh number; Gr: Grashof number

Prandtl number

$$Pr = \mu \frac{c_p}{\lambda}. \quad (5)$$

Nusselt number

$$Nu = \frac{\alpha \ell_0}{\lambda}, \quad (6)$$

Grashof number:

$$Gr = \frac{\Delta T \cdot g \cdot \beta \cdot l^3}{v^2}, \quad (7)$$

Where ΔT is temperature difference, g is the gravitational acceleration; β is the coefficient of volume expansion; l is determining size, v is the kinematic viscosity

Rayleigh number:

$$Ra = Gr \cdot Pr. \quad (8)$$

Then the similarity criterion equation will have the form:

$$\mathrm{Nu} = \mathrm{F}(\mathrm{Ra}). \quad (9)$$

Next, by calculating the Nusselt number according to (6) the value of the convective heat transfer coefficient will be obtained (α).

$$\alpha = Nu \cdot \frac{\lambda}{\ell_0}, \quad (10)$$

where l_0 = S/P (surface area to perimeter) — characteristic surface size of a multilayer system. In this problem l_0 = 0,214.

The instability of the natural convection process at surfaces of various shapes and placement in space has given rise to a wide variety of empirical formulas for the form of the criterion equation (9). In the local heat engineering literature, the criterion equations of Mikheev and Micheeva (1977) and Isachenko et al. (1981) are used to solve a similar problem posed by us in the work.

If, in case of forced convection, the results of calculating the Nusselt number according to the criterion equations of different authors are virtually identical, then in case of natural convection they differ by a significant 20–30 %.

According to Mikheev and Micheeva (1977), if the heated surface is facing upwards, then the movement proceeds according to the scheme shown in Figure 3 — a or Figure 3 — b.

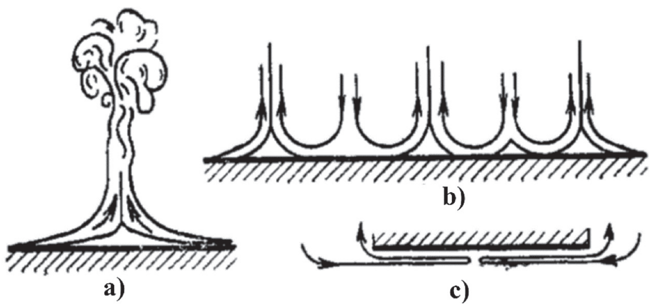
Due to the presence of a continuous flow of heated air from the edges, the central part of the surface turns out to be isolated. The free convection movement of air at the surface occurs only due to the influx of cold air from above (Figure 3 — b). If the heated surface is facing downwards, then in this case the air movement occurs only in a thin layer below the surface (Figure 3 — c); the rest of the air mass below this layer remains stationary.

Thus, the following heat exchange schemes are possible in the given task:

1. The air temperature is higher than the temperature of the multilayer system (product heating)

Then the upper surface (facing up) has free convection air movement according to the scheme (Figure 3 — c), and the lower surface (facing down) has unstable convection according to the scheme (Figure 3 — a or b).

Figure 3
Heat Transfer under Free Convection



2. The air temperature is less than the temperature of the multilayer system (product cooling)

The situation is opposite to the one described above. The upper surface (facing up) has unstable convection (Figure 3 — a or b), and the lower surface has stable convection (Figure 3 — c).

The stability and instability of the process were taken into account in the calculation of the Nusselt criterion number according to the corresponding criterion equations (11,12) proposed by Rohsenow et al. (1998), Fujii et al. (1972):

for an unstable process (Nu_1):

$$Nu_1 = \frac{0{,}560 \cdot Ra^{1/4}}{\left[1 + (\frac{0{,}492}{Pr})^{9/16}\right]^{4/9}}; \quad (11)$$

for a stable process (Nu_2):

$$Nu_2 = 0{,}58 \cdot Ra^{1/5}. \quad (12)$$

The unstable process proceeds under the condition of $T_a < T_p$ on the upper surface of the multilayer system, and at $T_a > T_p$ on the lower surface. A stable process takes place on the lower surface at $T_a < T_p$ and on the upper surface at $T_a > T_p$.

The results of the calculations of the criterion numbers are presented in Table 9.

The results of calculating the heat transfer coefficient (α_1, α_2) for each discrete temperature are presented in Table 10.

Thus, at the boundary with the ambient air, the upper and lower surfaces of the multilayer system have Newton-Richman boundary conditions. It is also assumed that there is an ideal heat contact between the layers inside the multilayer system and the values of temperature and density of heat fluxes at the boundary of the layers are equal at any given time.

Table 9

Calculated Values of the Criterion Numbers for the Task

T_a, °C	T_p, °C	T_{det}, °C	Gr·10E⁶	Ra·10E⁶	Nu_1	Nu_2
-35	5	-15.0	103.0	73.7	39.9	21.7
-30	5	-12.5	86.3	61.7	38.1	21.0
-25	5	-10.0	70.8	50.6	36.3	20.2
-20	5	-7.5	56.5	40.4	34.3	19.3
-15	5	-5.0	43.4	30.9	32.1	18.3
-10	5	-2.5	31.2	22.2	29.5	17.1
-5	5	0.0	19.9	14.2	26.4	15.6
0	5	2.5	9.6	6.8	22.0	13.5
5	5	5.0	0.0	0.0	0.0	0.0
10	5	7.5	8.8	6.3	21.5	13.3
15	5	10.0	17.0	12.0	25.3	15.1
20	5	12.5	24.5	17.4	27.7	16.3
25	5	15.0	31.4	22.2	29.5	17.1
30	5	17.5	37.7	26.7	30.9	17.7
35	5	20.0	43.5	30.8	32.0	18.2

Where Nu_1, Nu_2 — the Nusselt number for unstable and stable convection, respectively; T_{det} — determining temperature

Table 10

The value of the Yeat transfer Coefficient for Each Discrete Temperature

T_a, °C	T_p, °C	$α_1$	$α_2$
	$T_a < T_p$		
-35	5	4.30	2.34
-30	5	4.15	2.28
-25	5	3.98	2.21
-20	5	3.79	2.13
-15	5	3.57	2.04
-10	5	3.32	1.92
-5	5	2.99	1.77
0	5	2.51	1.54
	$T_a > T_p$		
10	5	1.54	2.49
15	5	1.76	2.96
20	5	1.91	3.26
25	5	2.02	3.50
30	5	2.12	3.69
35	5	2.19	3.85

Where T_a — Ambient temperature (BC), T_p — the initial temperature of the product (IC), $α_1$ — the heat transfer coefficient at the upper surface (BC), $α_2$ — the heat transfer coefficient at the bottom surface (BC).

Limits of Applicability of the Model

Geometric shape. Laying a parallelepiped, we believe that the error will be small for the actual shape, but deviations are possible.

A free approach to calculating the heat transfer coefficient. Our assumptions are quite loose, since there is no enough model for free convection, we choose the recommendations of some authors, according to other authors there will be deviations.

The possibility of phase transitions. The model does not take into account possible phase transitions (freezing) of the matter. We consider ambient temperatures presumably up to possible phase transitions of matter, since phase transitions change the physical properties of the medium (density, specific heat capacity and thermal conductivity coefficient, etc.).

Mathematical Formulation of the Task

In accordance with the introduced approximations and assumptions, the following mathematical formulation of the task is posed.

The analytical description of the thermal conductivity process includes a system of differential equations, initial conditions for the problem parameters and boundary conditions. The differential equation of thermal conductivity in the absence of internal heat sources has the form:

$$\frac{\partial T}{\partial \tau} = α\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right). \quad (13)$$

1. Initial conditions T (τ = 0) — the temperature of the product at the initial time
2. Boundary conditions of type 1–4

The shape and geometric dimensions of the object are important here.

Differential equations together with the so-called unambiguity conditions (initial conditions and boundary conditions) form a complete mathematical formulation of the task. That is, knowing the geometric shape of the object under study, the initial and boundary conditions, it is possible to solve the differential equations to the end and find the temperature function $T(x, y, z, τ)$.

To formulate a mathematical problem corresponding to a physical model, the thermal conductivity equation (13) will be considered one-dimensional (variable X), boundary conditions on the upper and lower surfaces of the first and last layers of type 3, and conditions at the interface of other layers of type 4.

The set problem has the following form:

$$\begin{cases} \frac{\partial T_1}{\partial \tau} = α_1 \frac{\partial^2 T_1}{\partial x^2} \\ \frac{\partial T_2}{\partial \tau} = α_2 \frac{\partial^2 T_2}{\partial x^2} \\ \frac{\partial T_3}{\partial \tau} = α_3 \frac{\partial^2 T_3}{\partial x^2} \end{cases}. \quad (14)$$

Initial conditions:

$$\begin{cases} \tau = 0 \\ T_1 = T_2 = T_3 = T_p \end{cases}, \quad (15)$$

where T_p — initial temperature of the product.

Boundary conditions:

$$\begin{cases} x = 0 & \lambda_1 \left(\frac{\partial T_1}{\partial x}\right) \Big|_{x=0} = \alpha_1 (T_1 \big|_{x=0} - T_{\text{в}}) \\ x = H & -\lambda_3 (\frac{\partial T_3}{\partial x}) \Big|_{x=H} = \alpha_2 (T_3 \big|_{x=0} - T_{\text{в}}) \end{cases}$$

$$1 \leq \acute{\imath} \leq 13$$

$$\begin{cases} T_1 \big|_{x\acute{\imath}} = T_2 \big|_{x\acute{\imath}} & \acute{\imath} \in [1, 12] \\ \lambda_1 \left(\frac{\partial T_1}{\partial x}\right) \Big|_{x\acute{\imath}} = \lambda_2 \frac{\partial T_2}{\partial x} \Big|_{x\acute{\imath}} \end{cases} \quad . \qquad (16)$$

$$\ldots$$

$$\begin{cases} T_1 \big|_{x_{13}} = T_3 \big|_{x_{13}} \; \acute{\imath} = 13 \\ \lambda_1 \left(\frac{\partial T_1}{\partial x}\right) \Big|_{x\acute{\imath}=13} = \lambda_3 \frac{\partial T_3}{\partial x} \Big|_{x\acute{\imath}=13} \end{cases}$$

Analytical Solution of the Thermal Conductivity Problem for a Multilayer System

To solve the multilayer problem (17–19), we used the analytical solution presented in (Fuji and Imura, 1972) and adapted it to the conditions of heat transfer with SCM in transport packaging under ambient conditions. The principle of superposition states that the solution can be represented as:

$$\mathrm{T}_i(x,t) = \sum_{n=1}^{\infty} \Gamma_n(t) R_{i,n}(x) + q_i(x,t), \qquad (17)$$

where

$T_i(x, t)$ — temperature of the layer;

$R_{i,n}(x)$ — eigenfunctions of the layer having the property of orthogonality to each other;

$\Gamma_n(t)$ — the temporary part for the eigenfunction, has the property of independence from the layer;

$q_i(x, t)$ — an auxiliary function homogenizing the boundary conditions (16) of the form:

$$q_i(x,t) = \begin{cases} T_B \alpha_1 \frac{\sigma - x}{\alpha_1 \sigma + \lambda_1}, & i = 1 \\ 0, \; i = 2..13 \\ T_B \alpha_3 \frac{\mathrm{x} - \mathrm{H} + \mathrm{l}}{\alpha_3 \mathrm{l} + \lambda_3}, & i = 14 \end{cases} \quad . \qquad (18)$$

The type of auxiliary function (18) is due to the fact that the temperature at the ends is static, therefore there is no dependence on time.

The eigenfunctions of the solution have the form:

$$R_{i,n}(x) = \mathrm{C}_{i,n} \sin(\Lambda_{i,n} x) + D_{i,n} \cos(\Lambda_{i,n} x)$$

$$\Lambda_{i,n} = \Lambda_{1,n} \sqrt{\frac{\alpha_1}{\alpha_i}} \quad , \qquad (19)$$

where is proper numbers. The search for coefficients и is carried out by calculating the determinant of the system of equations for eigenfunctions (Antonopoulos and Tzivanidis, 1996).

General view of the temporary part $\Gamma_n(t)$:

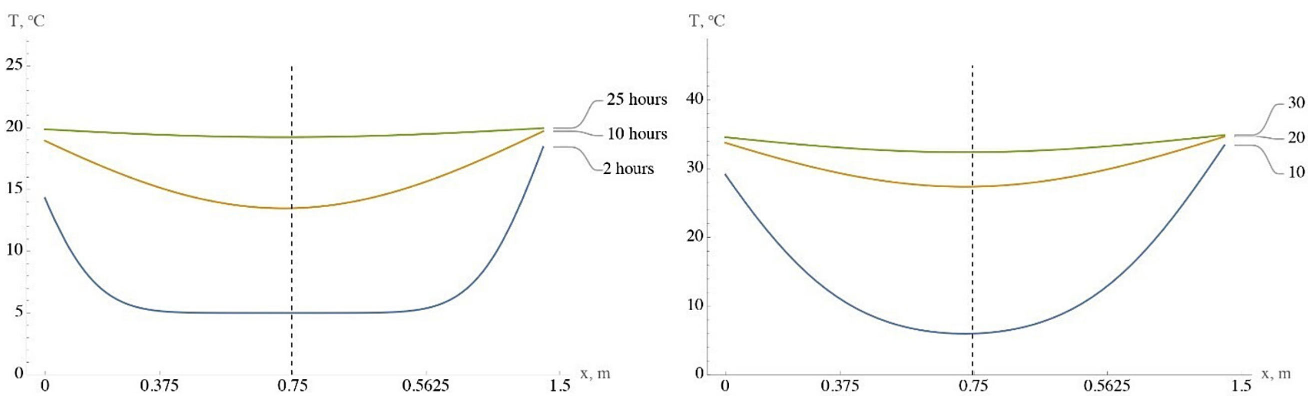
$$\Gamma_n(t) = f_n^* e^{-a_1 \Lambda_{1,n}^2 t}$$

$$f_n^* = \frac{1}{N_n} \sum_{i=1}^{14} \frac{\lambda_i}{a_i} \int_{x_{i-1}}^{x_i} [T_\Pi - q_i(x,0)] R_{i,n}(x) dx \qquad (20)$$

$$N_n = \sum_{i=1}^{14} \frac{\lambda_i}{a_i} \int_{x_{i-1}}^{x_i} {R_{i,n}}^2(x) dx$$

Figure 4

Temperature Profile of the Multilayer System with SCM for Initial Temperatures $T_p = 5$°C and (a) $T_a = 20$°C, (b) $T_a = 35$°C



In general, the function has a more complex form, but since the expression for air temperature (14–16) does not depend on time, the Laplacian and the time derivative of the auxiliary functions are zeroed and do not participate in the summation when constructing . At the same time, it is possible to generalize the solution to fluctuating air temperature indicators that simulate changes in daytime temperature or temperature during transportation.

The solution of equation (17) is shown in Fig. 4. The software solution developed for the implementation of equations (17–20) makes it possible to obtain temperature profiles for arbitrary initial temperatures in the studied range from minus 35 °C to 35 °C.

To calculate this problem, the summation used the first eigenfunctions corresponding to the eigenvalues to achieve a total error from the true solution not exceeding 5 %. In most cases it was equal to 3.

The figure shows that the influence of corrugated layers and a layer of wood on the solution, although minimal, can still be observed a displacement of the profile from symmetry to the side where there are more conductive layers. Also, the analytical solution of the problem made it possible to determine the duration of the temperature change of the object under study, which was 36.7 hours at maximum temperature pressures for the range from 5°C to 35°C and 41.1 hours for the range from 5°C to minus 35°C.

DISCUSSION

Given the rapid advancement of digital technologies, which are an integral part of the industrial revolution—Industry 4.0—various sectors of the economy (such as construction, automotive manufacturing, and the textile industry) are undergoing large-scale digitalization of processes to optimize operations and improve efficiency (Nagar & Sreenivasa, 2024). One of the most widely used digitalization tools is mathematical process modeling (Medennikov & Raikov, 2020; Erdogdu, 2023). Researchers worldwide develop models of objects, equipment, and processes, which are subsequently tested under real-world conditions to evaluate the effectiveness of their developments (Hu et al., 2013; Destro et al., 2021; Bunta et al., 2023).

In our study, we conducted heat transfer modeling of SCM with the ambient air to apply the developed model in predicting the duration of temperature changes during transportation. Several major assumptions were made to simplify the final model, including neglecting the geometry of metal cans containing SCM inside the transport package, excluding phase transitions, omitting air gaps inside the cartons with the product, disregarding convective flows within the product, and neglecting heat exchange through the side walls of the studied system.

A comparison with two-dimensional heat conduction models, such as the spherical coordinate model presented in (Jain & Singh, 2010), demonstrated that by selecting a sufficient number of terms in the time series Γn (t), the one-dimensional layered model can approximate them with arbitrary accuracy for the investigated configuration of SCM cans. Regarding the exclusion of convective flows within the product, Rao & Anantheswaran (1988) demonstrated that convective heat transfer significantly influences the temperature profiles of low-viscosity liquid products, such as juices or broths. However, for high-viscosity products like SCM, convective flows are minimal, and heat transfer occurs primarily through conduction, which is also confirmed in (Kumar et al., 1990).

In Friso's (2015) study on modeling heat processes in canned products, factors related to convection within the product and the complex geometry of the cans were also excluded, as the entire approach focused on analyzing the slowest heating point in the can — a key criterion for evaluating sterilization efficiency. However, this approach is not fully applicable to our study, as we are interested not only in the temperature in the slowest heating point but also in its distribution across the entire system at different moments in time. Furthermore, Friso (2015) highlights the significant computational effort required when using numerical methods to solve heat transfer problems, leading him to prefer an analytical solution—an approach also chosen in our study.

For example, in (Antonopoulos & Tzivanidis, 1996), a more general analytical approach is applied to solving the one-dimensional heat conduction problem under convective boundary conditions. Therefore, adapting the analytical solution from (Biswas & Singh, 2015) appeared more promising, as the authors demonstrated the possibility of avoiding explicit computation of the auxiliary function under conditions similar to SCM storage conditions, significantly reducing computational requirements.

Thus, the assumptions introduced in the model and the modeling approaches applied in this study align with solutions used by other heat transfer researchers. The distinguishing feature of our research is that the combination of selected methods allowed us to develop a tool for solving practical problems in SCM transportation (a strategically important food product) while ensuring computational efficiency.

CONCLUSION

In this study, the applicability of a one-dimensional heat transfer model for predicting the duration of SCM temperature changes over an extended temperature range was established. This approach reduces computational effort compared to modeling in a multidimensional space.

The primary outcome of the research is the development of a one-dimensional modeling approach for SCM heat transfer with the ambient air, including its formulation and the creation of the model itself. Additionally, the possibility of formulating an analytical form of equations for the temperature of individual layers was demonstrated. The mathematical framework investigated allows for extending the solution to scenarios with fluctuating ambient temperatures, and the obtained results, in general, enable the validation of the developed model and its comparison with empirical studies on SCM.

A limitation of the proposed approach is the exclusion of phase transitions, which could be addressed in future improvements to the method. Demonstrating the effectiveness of the developed model, which is a logical next step, could expand the range of specialized business software solutions for logistics planning, transportation cost estimation, and accounting for ambient conditions during shipment. Furthermore, the proposed solution can be adapted for other food products.

AUTHOR CONTRIBUTIONS

Ekaterina I. Bolshakova: investigation, methodology, draft manuscript preparation, manuscript writing and editing, visualization, data administration, project administration.

Sergey V. Motylev: investigation, methodology, data verification, draft manuscript preparation, formal analysis.

Vladislav K. Semipyatny: investigation, methodology, draft manuscript preparation, formal analysis.

Aleksandr G. Kruchinin: methodology, resources, manuscript writing and editing, research supervision, project administration.

Svetlana N. Turovskaya: data administration, manuscript writing and editing.

Elena E. Illarionova: resources, manuscript writing and editing.

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